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MODELING OF HEAT AND MASS TRANSFER AND AERODYNAMICS IN A BULK OF STORED AGRICULTURAL RAW MATERIAL

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Methods of mathematical and computer simulation of the microclimate in storehouses for juicy agricultural raw material are suggested. The calculation algorithms developed allow one to determine the aerodynamic and temperature–moisture characteristics with account for natural convection and arbitrary shape of storehouses in order to optimize the storing of biological products.

Introduction. To ensure the preservation of biological products, of greatest importance is the temperature–moisture regime in storehouses and in the products themselves. It should be kept in mind that in some storehouses not only are the needed temperature–moisture parameters of the medium maintained but the composition of the latter is also altered. One can ensure the maintenance of the parameters, specified by technology, in storehouses and in the mass of raw material only by arranging simultaneous functioning of the systems of ventilation, heating (cooling), and wetting (drying). The required running parameters of microclimate in storehouses depend on a great number of factors, many of which are variable due to the physiological changes in the biological medium with time and to varying temperature, moisture content, velocity, and pressure of outdoor air, etc. [1–4].

Capital storehouses for juicy agricultural raw material have a capacity from 500 to 10,000 tons or more. Along with capital storehouses, when the latter are insufficient in number or are absent, assembled–disassembled structures are widely used for storing potatoes and vegetables with active ventilation; they are 1–2 orders of magnitude less expensive. One such type of storehouse which combines a relatively low cost, ease of fabrication and mounting, and availability of the materials used, as well as high preservation and quality of products, are large-size storage piles [1, 5]. They can hold up to 400 tons of potatoes or 320 tons of fodder beat. The elements of the constructions of piles can be fabricated at small factories and be mounted in 2–3 days. A ventilation module [1] is installed in the central part of the structure. Thus, under any weather conditions ventilation air of a given temperature is fed into the mass of products using the heat released from the potatoes and vegetables stored. The height of the mound is up to 4 m (Fig. 1). The products are covered by a multilayered coating consisting, say, of bales of pressed straw and film material. During the service life of such storehouses, data are accumulated that allow one to perfect their construction.

One of the reasons for the high losses of products in the process of storing is the inadequate attention devoted to heat- and moisture-exchange processes in the raw material mound and the inadequacy of heat-engineering methods of calculation of storehouses. The aim of the present study is the development of methods of heat-engineering calculation of vegetable and potato storehouses by solving three-dimensional conjugate problems of nonstationary heat and moisture transfer for storehouses of various shapes and capacity.

Mathematical Model. The difficulties of mathematical simulation of heat and moisture exchange in pills are caused by their shape greatly differing from a rectangular one. Here, one must, to full measure, model the aerodynamics of the mound both in the case of forced convection (active ventilation) and natural convection. Moreover, the general problem of heat and moisture transfer should be formulated as a three-dimensional, nonstationary one.

It should be noted that there are virtually no works on modeling natural convection in stored products, even for the two-dimensional case, or else simple mathematical models are used for calculations [3, 4, 6, 7]. This is due to the computational problems.

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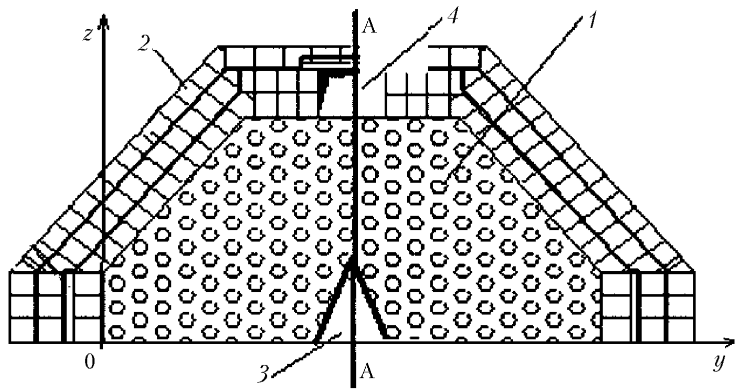

Fig. 1. Schematic of a large-size pile: 1) mound of a product; 2) covering; 3) intake air pipeline; 4) upper air pipeline; AA, symmetry axis.

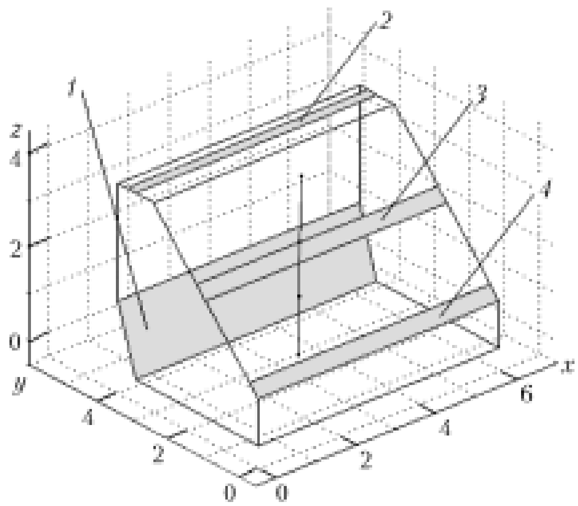

Fig. 2. Calculation scheme of a pile (symmetry planes $x = 6$ and $y = 4$; mound ($0 < x < 6$, $0 < y < 4$, $0 < z < 4$)): 1) intake air pipeline ($3.5 < y < 3.75$, $0 < z < 1.5$); 2–4) exit channels: 2) ($3.5 < y < 3.75$, $z = 4$); 3) $1.5 < y < 1.75$, $1 < z < 1.25$); 4) ($0 < y < 0.25$, $1 < z < 1.25$).

The majority of researchers formulated mathematical models with a number of simplifications: the inconjugacy of the thermal problem (the temperature of the enclosing structures is known and it is independent of the thermal state of the pile), one-dimensionality, stationarity, and the uncombined heat and moisture transfer. However, the thermal and moisture state of biological products in a storehouse depends not only on the thermodynamic processes occurring in the pile but also on outer effects acting through the storehouse structures (this especially concerns the periphery part of the pile), as well as on thermal perturbations introduced by the equipment. The totality of the specific features can be allowed for only by a mathematical model of coupled heat transfer based on the conjugate nonstationary problem of heat and moisture transfer for the entire structure–mound system in their thermal interrelation rather than separately in each of the regions involved [8].

In order to determine the basic temperature–moisture parameters of stored agricultural raw material it is necessary to formulate a three-dimensional problem for a storehouse of arbitrary shape on the basis of the experience accumulated to date in modeling one- and two-dimensional problems [8–11]. The calculation scheme of a pile is presented in Fig. 2. From an intake air duct 1 ventilation air of given temperature and moisture content enters the mound with a certain velocity. The main aim of such ventilation is to prevent self-heating of the mound by the heat released by biological products. After being passed through the mound the air is removed through exit channels 2–4.

The determination of the structural features of the storehouse (shape, position of the channels and of the heat-insulating coating, etc.) and ventilation regimes are the basic problems of computational experiments on the basis of mathematical models for storage optimization. In formulating a mathematical model of heat- and moisture transfer in a storehouse the air is considered incompressible, since it moves with a velocity not exceeding 0.7–0.8 m/sec, whereas the pressure drop (hydraulic resistance of the mound) is 120 Pa/m, and the porosity is taken to be constant. Then the system of equations of heat and mass transfer is formulated as follows [1, 12, 13]:

the thermal-conductivity equation for the coating with account for the outdoor air infiltration

$$\frac{\partial T_{\text{cov}}}{\partial \tau} + W_{\text{cov}} \frac{\rho_a c_a}{(\rho c)_{\text{cov}}} \frac{\partial T_{\text{cov}}}{\partial n} = a_{\text{cov}} \Delta T_{\text{cov}} , \quad \Delta \text{ — is the Laplace operator ;} \tag{1}$$

the energy equation for a mound of products

$$\frac{\partial T_m}{\partial \tau} = \frac{1}{c_m} q_0 \exp (bT_m) - \frac{\beta q_e F_m \varepsilon_m E}{\rho_m c_m} (f (T_m) - d) + \text{div} (a_m \text{ grad } T_m) - k_1 (T_m - T_a) , \tag{2}$$

the term $\frac{\beta q_e F_m \varepsilon_m E}{\rho_m c_m}(f(T_m) - d)$ on the right-hand side of Eq. (2) determines heat transfer from the surface of bulbs due to evaporation up to saturation of the surrounding air with moisture, $\frac{1}{c_m} q_0 \exp (bT_m)$ denotes biological heat releases from the products, $k_1 = \frac{\alpha_c F_m}{\rho_m c_m}$;

the energy equation for air in the mound of the products

$$\frac{\partial T_a}{\partial \tau} + \mathbf{u} \cdot \text{grad } T_a = \text{div} (a_a \text{ grad } T_a) + k_2 (T_m - T_a) , \quad k_2 = \frac{\alpha_c F_m}{\varepsilon \rho_a c_a} ; \tag{3}$$

the equation of moisture diffusion

$$\frac{\partial d}{\partial \tau} + \mathbf{u} \cdot \nabla d = \frac{D}{\varepsilon} \Delta d + \frac{\beta F_m \varepsilon_m E}{\rho_a \varepsilon} (f (T_m) - d) , \tag{4}$$

the source term $\frac{\beta F_m \varepsilon_m E}{\rho_a \varepsilon}(f(T_m) - d)$ in Eq. (4) determines moisture transfer between the bulbs and the ventilation air;

the equations of motion and continuity for air in the mound which are written for the case of filtration in large-cell media [9, 14, 16]:

$$\frac{\partial \mathbf{u}}{\partial \tau} + (\mathbf{u} \cdot \nabla) \cdot \mathbf{u} = - \frac{1}{\rho_a} \text{grad } P - \frac{\mathbf{u}}{\rho_a u} F (u) - \mathbf{k} \beta_0 g (T_a - T_m) , \tag{5}$$

$$\text{div } \mathbf{u} = 0 . \tag{6}$$

The initial conditions are: $T_{\text{cov}}(x, y, z, 0) = T_{\text{cov}0}$, $T_a(x, y, z, 0) = T_{a0}$, $T_m(x, y, z, 0) = T_{m0}$, $d(x, y, z, 0) = d_0$, $\mathbf{u}(x, y, z, 0) = \mathbf{u}_0$.

The basic boundary conditions are:

on the outer surfaces of enclosures

$$- \lambda_{\text{cov}} \frac{\partial T_{\text{cov}}}{\partial n} = \alpha_m (T_{\text{cov}} - T_{\text{out}}) + \alpha_r (T_{\text{cov}}^4 - T_{\text{out}}^4) , \tag{7}$$

on the mound surface open for the exit of the ventilation air

$$P = 0\,, \quad \text{grad}\, T_a = \nabla d = 0\,, \tag{8}$$

the conditions on symmetry planes

$$\frac{\partial T_m}{\partial n} = \frac{\partial T_a}{\partial n} = \frac{\partial d}{\partial n} = \frac{\partial \mathbf{u}}{\partial n} = 0\,, \tag{9}$$

the conditions at $z = 0$, the floor surface

$$T_m \big|_{z=0} = T_n\,, \quad \text{grad}\, T_a = \nabla d = 0\,, \quad \frac{d\mathbf{u}}{dn} = 0\,, \tag{10}$$

at the exit from the ventilation channel

$$\mathbf{u}\cdot\mathbf{n} = u_1\,, \quad d = d_1\,, \quad T_a = T_m = T_{ch}\,, \tag{11}$$

heat exchange at the mound–coating interface (conjugation conditions)

$$T_{cov} = T_m\,, \quad \lambda_{cov}\frac{\partial T_{cov}}{\partial n} = \lambda_m \frac{\partial T_m}{\partial n}\,. \tag{12}$$

The relations $f(T_m) = A + BT_m$ represent approximation of the dependence of an equilibrium moisture content of air on temperature [1], $A = 0.00371$, $B = 0.00031$ for the temperature interval admissible in storing; $E = 161{,}332$ Pa; $\rho_m = (1 - \varepsilon)\rho_p$ is the density of the mound; $F(u)$ is the aerodynamic resistance depending on the Reynolds number and the porous properties of the mound [14]:

$$F(u) = \frac{F_m^2 \mu K}{\varepsilon^3} u + \frac{F_m \rho_a K_i}{2\varepsilon^3} u^2\,. \tag{13}$$

An important physical characteristic of the process of heat transfer between the product stored and ventilation air is the heat-transfer coefficient α_c. Numerous investigations carried out by various researchers give different values for this coefficient. Our calculations and comparison with experimental data show that in this case the most appropriate is the formula [15]

$$\alpha_c = 0.05 d_p + \frac{7.27 u^{0.67}}{d_p^{0.33}}\,.$$

It was assumed [1, 5, 7] that $\beta = 0.65\cdot10^{-7}\alpha_c$ and $D = D_0 \dfrac{0.101}{P}\left(\dfrac{T_a}{273}\right)^{1.81}$.

Numerical Solution and Results. In order to solve problems (1)–(12), we employed the methods of finite elements and finite differences [1, 6, 16, 17] using the POSOKh software developed by us [10]. The general boundary-value problem is solved in POSOKh following the scheme of splitting multidimensional equations into one-dimensional ones and thereafter by a locally one-dimensional method from an implicit-difference scheme [11]. Based on the large number of computational and natural experiments carried out [1, 11, 18], we may state that the method used is practically stable toward the choice of the number of grid steps along both coordinates, as well as toward the step magnitude in time. Unfortunately, the finite-difference methods are efficient only for regions of rectangular shape. Therefore, for the piles considered in the present work, the POSOKh was complemented with a finite-element algorithm (FEA) which allows one to perform calculations for regions no matter how complex and "sandwiched" they are [1, 17]. The conjugation conditions (12) are satisfied in this case automatically.

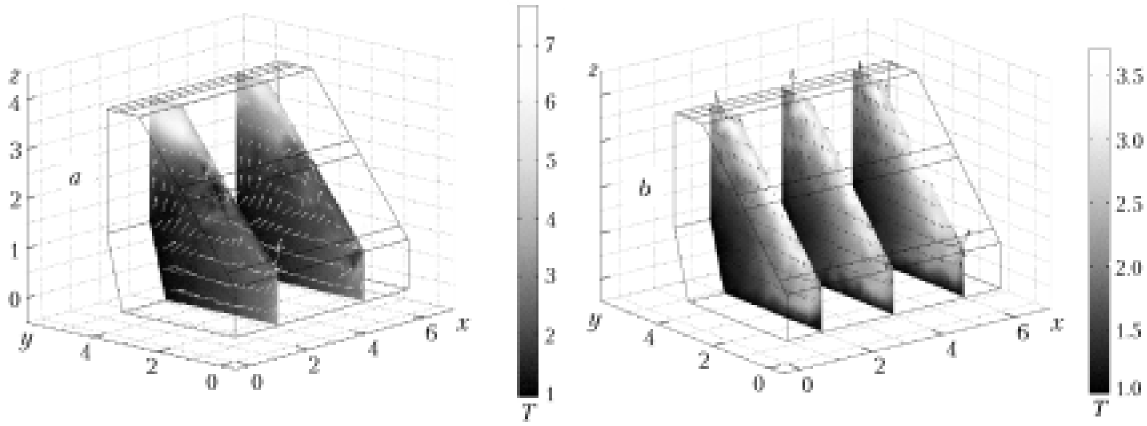

Fig. 3. Distribution of temperature (grey tints) and velocities (arrows) in characteristic sections of a mound in a steady state [a) exit channels 3 and 4; b) exit channels 2 and 4 — Fig. 2].

The calculations were carried out on grids of different fineness using the Lagrangian-type triangular elements and quadratic approximation. In order to control the results of computations, a posteriori estimates of the accuracy of *m*-nodal finite-element approximations were used [1, 19, 20].

A characteristic feature of the technique proposed for the problems studied is the possibility of modeling for considerable time intervals, that is, 100 h or more (due to the character of the phenomena considered). For this purpose, at the first stage we solve a linearized problem to obtain the best "zero" approximation to a nonlinear one. The triangular or tetrahedral unstructured grid is determined automatically by the grid generator. The adaptive regime of grid generation minimizes the computational error due to the redetermination of the grid in accordance with problem solution. The methods to solve the Cauchy problem obtained after ensembling are selected automatically to determine the maximum step in time without loss in the accuracy of the calculations.

As a characteristic example, in the present work we consider a mound of potatoes (Figs. 1 and 2) of width 8 m (by virtue of symmetry only half of the width (4 m) is shown in the figures), height 4 m, and length 12 m (by virtue of symmetry only half of the length (6 m) is shown in the figures). The numerical values of the principal physical constants [1, 15] are: $b = 0.0671$ 1/K, $q_0 = 0.01$ W/kg, $d_{\rm p} = 0.052$ m, $\varepsilon_{\rm m} = 0.012$, $\lambda_{\rm m} = 0.52$ W/(m·K), $\rho_{\rm m} = 1080$ kg/m^3, $c_{\rm m} = 3560$ J/(kg·K), $F_{\rm m} = 120.5$ m^2/m^3, $q_{\rm e} = (597 - 0.55T_{\rm a})\cdot 4.19\cdot 10^3$ J/kg, $\varepsilon = 0.4$, the von Karman–Kozeny constant is $K = 4.7$ and and the so-called "inertial" constant $K_{\rm i} = 0.75$.

The moisture content of air at the entrance to the mound was taken equal to $d = 0.004$ kg/kg and the temperature to $T_{\rm a} = 1^{\rm o}$C (the relative moisture content $\Phi = 98\%$). The ventilation air velocity at the entrance to the mound was $u_1 = 0.12$ m/sec. The daily mean temperature of the outdoor air $T_{\rm out}(x, y, z, \tau) = -10^{\rm o}$C. The temperature acceptable for storing in a mound is considered to be $T_{\rm m} = (1\text{–}4)^{\rm o}$C [1].

Figure 3 presents the results of calculation of a developed thermal state of the mound for different dispositions of exit channels, under other conditions being identical. A comparison of Fig. 3a and b demonstrates the strong effects that can be exerted by seemingly insignificant structural changes on the preservation of biological products. The distribution of temperature (grey tints) and velocity (arrows) in the characteristic sections of the pile in Fig. 3a shows that in a portion of the pile the temperature of $4^{\rm o}$C admissible for the preservation of the product was exceeded (to the right of the figure the temperature scale is given with marked minimum and maximum values of temperature). This is especially evident in the upper portion of the mound. Figure 3b depicts a normal temperature regime with $1^{\rm o} < T_{\rm m} < 3.72^{\rm o}$C. These differences are explained by the fact that with the exit channels positioned as shown in Fig. 3b the ventilation air arrives in sufficient quantity in unit time to self-heating sites and fulfills its main function in full measure, viz., to ensure the temperature and moisture content needed for normal storage.

In order to determine the time allowable by definition for repairing electrical equipment during which the preservation of the products is ensured despite the switching-off of the entire energy supply system, the necessity of modeling the development of natural convection in a mound of agricultural raw material after turning-off of the system of active ventilation arises. Our calculations show that 32 h is the time upon lapse of which the temperature at the characteristic points of the mound (the position of these points on the straight line inside the pile is shown in Fig. 2) passes into the upper admissible boundary of 4^oC for the exit holes located as shown in Fig. 3b. The moisture content of air in the mound increases with the temperature of the product virtually proportionally to the rise in temperature.

If we compare the results obtained with the calculations that disregard natural convection [11], we may see that the error in determining the time for admissible switching-off of the systems that supply power to storehouses may amount to up to 15–20% without account for natural convection.

Comparison of the results of calculations of basic heat-engineering parameters that determine the preservation of products by a two-dimensional model with the results given by a three-dimensional one has shown that the quantitative disagreement between them reaches 7–12%.

Conclusions. The structural characteristic features of storehouses for agricultural products are often responsible for the microclimate in them and, consequently, they dictate the choice of the storage technology. Modeling of a microclimate with allowance for the natural characteristic features of a mound of products is possible to a full measure only on the basis of a three-dimensional nonstationary problem of interrelated heat and mass transfer. The mathematical model suggested in the present work allows one to carry out an analysis of the temperature and moisture content of stored biological products with allowance for various structural features of storehouses for agricultural products. The examples given illustrate the possibilities of such modeling based on earlier tested numerical solutions. We have developed programs that make it possible to rather rapidly and simply determine the velocities, temperatures, and moisture contents in different zones of the mound depending on varying outdoor conditions and variable thermophysical characteristics.

NOTATION

a_a, thermal diffusivity of air in the mound with account for porosity, m^2/sec; a_{cov}, thermal diffusivity of a covering material, m^2/sec; a_m, thermal diffusivity of a mound with account for porosity, m^2/sec; b, temperature respiratory coefficient, 1/K; c_a, heat capacity of air, J/(kg·K); c_m, heat capacity of the mound, J/(kg·K); c_{cov}, heat capacity of covering with allowance for porosity, J/(kg·K); D, coefficient of diffusion of steam in air, m^2/sec; d, moisture content of air in a mound, kg/kg; d_0, initial humidity of air in the mound, kg/kg; d_1, humidity of air at the entrance to the mound, kg/kg; d_p, mean diameter of the physical element of a product, m; E, conversion factor relating moisture content to the partial pressure (elasticity) of steam, Pa; $f(T_m) = A + BT_m$, approximation of the dependence of equilibrium moisture content of air on temperature, kg/kg; F_m, specific surface of a mound, m^2/m^3; g, gravity acceleration, m/sec^2; K, von Karman–Kozeny constant; K_i, inertial constant; $\mathbf{k}$, unit vector of the OZ axis; $\mathbf{n}$, unit vector of the normal to the surface; n, normal to the surface; P, pressure in a mound, Pa; q_0, specific power of heat release from products at a temperature of 273 K, W/kg; q_e, specific heat of evaporation, J/kg; T_{cov}, temperature of covering, K; T_{cov0}, initial value of the temperature of covering, K; T_a, air temperature in a mound, K; T_{a0}, initial value of air temperature in a mound, K; T_m, temperature of the mass of products mounded, K; T_{m0}, initial value of the temperature of products, K; T_{out}, temperature of the outdoor medium, K; T_{ch}, air temperature at the inlet to a mound (at the outlet from ventilation channels), K; $\mathbf{u}$, air velocity in a mound, m/sec; u, absolute value of air velocity in a mound, m/sec; $\mathbf{u}_0$, initial air velocity in a mound, m/sec; u_1, air velocity at the entrance to a mound, m/sec; W_{cov}, rate of air filtration through a covering, m/sec; α_c, coefficient of convective heat transfer between the mound elements and ventilation air, $W/(m^2 \cdot K)$; α_m, coefficient of convective heat exchange between the outer surface of the enclosure and surrounding medium, $W/(m^2 \cdot K)$; α_r, coefficient of radiative heat exchange between the outer surface of the enclosure and the surrounding medium, $W/(m^2 \cdot K^4)$; β, moisture transfer coefficient, $kg/(m^2 \cdot Pa \cdot sec)$; β_0, coefficient of volumetric expansion, 1/K; ε, porosity of a mound; ε_m, evaporative ability of the elements of a mound; λ_a, thermal conductivity of air, W/(m·K); λ_{cov}, thermal conductivity of covering, W/(m·K); λ_m, thermal conductivity of a mound, W/(m·K); μ, dynamic viscosity of air, Pa·sec; ρ_a, air density, kg/m^3; ρ_p, density of a product, kg/m^3; ρ_{cov}, density of a covering with account for porosity, kg/m^3; ρ_m, density of a mound, kg/m^3; τ, time, sec; Φ, relative moisture content, %. Subscripts: 0, initial value; a, air; c, convective; ch, ventilation channel; cov, covering; e, evaporation; i, inertial; m, mound; n, normal; out, outdoor; p, product; r, radiative.

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